Intro Video: Section 5.3
The Fundamental Theorem of Calculus, Part 2

Math F251X: Calculus 1

FTC 1 says if $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$.

The Fundamental Theorem of Calculus, Part 2:

If f is continuous on $[a,b]$ and $F(x)$ is <u>any</u> antiderivative of f, then

$$\int_a^b f(x)\,dx = F(b) - F(a) = \boxed{F(x)\Big|_a^b}$$

<u>Example</u>: $\int_1^3 x^2\,dx$ Let $F(x) = \frac{x^3}{3}$. this is an antiderivative of x^2.

$$\int_1^3 x^2\,dx = \frac{x^3}{3}\Bigg|_1^3 = \frac{(3)^3}{3} - \frac{(1)^3}{3} = \frac{9}{3} - \frac{1}{3} = \frac{8}{3}$$

Example

① $$\int_3^{2\pi} \boxed{\sin(x) + e^x}\, dx = \left(-\cos(x) + e^x\right)\Big|_3^{2\pi}$$

$$= \left(-\cos(2\pi) + e^{2\pi}\right) - \left(-\cos(3) + e^6\right)$$

$$= -1 + e^{2\pi} + \cos(3) - e^6$$

② $$\int_1^3 \frac{y^3 - 2y^2 - y}{y^2}\, dy = \int_1^3 y - 2 - \frac{1}{y}\, dy$$

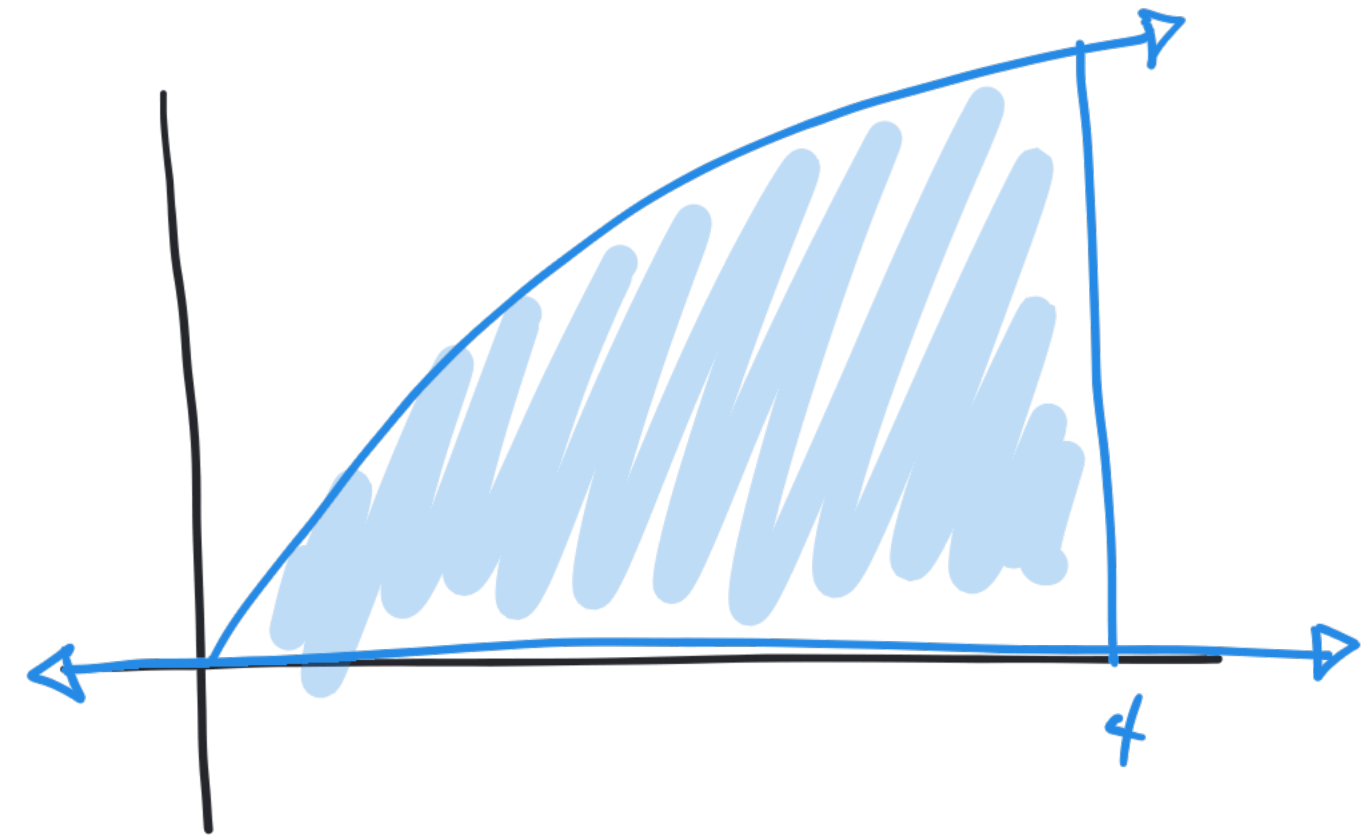
$$= \left(\frac{y^2}{2} - 2y - \ln|y|\right)\Big|_1^3$$

$$= \left(\frac{3^2}{2} - 2(3) - \ln(3)\right) - \left(\frac{1}{2} - 2 - \ln(1)\right)$$

$$= \frac{9}{2} - 6 - \ln(3) - \frac{1}{2} + 2 + 0 = 4 + 2 - 6 - \ln(3) = -\ln(3)$$

Example: Determine the area enclosed by

$$y = \sqrt{x}, \quad y = 0, \quad x = 4.$$



$$\text{Area} = \int_0^4 \sqrt{x}\, dx$$

$$= \int_0^4 x^{1/2}\, dx$$

$$= \frac{2}{3} x^{3/2} \Big|_0^4 = \frac{2}{3}(4)^{3/2} - 0 = \frac{2}{3}\left(\sqrt{4}\right)^3 = \frac{2}{3}(2)^3$$

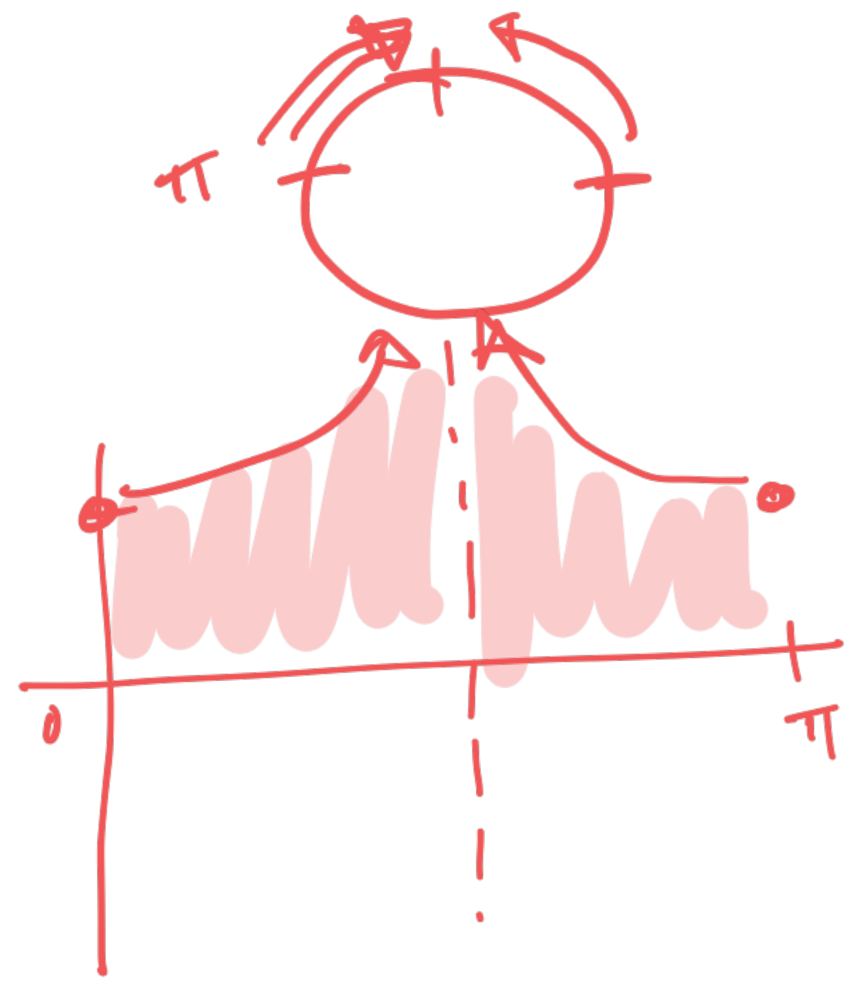
$$= \frac{16}{3}$$

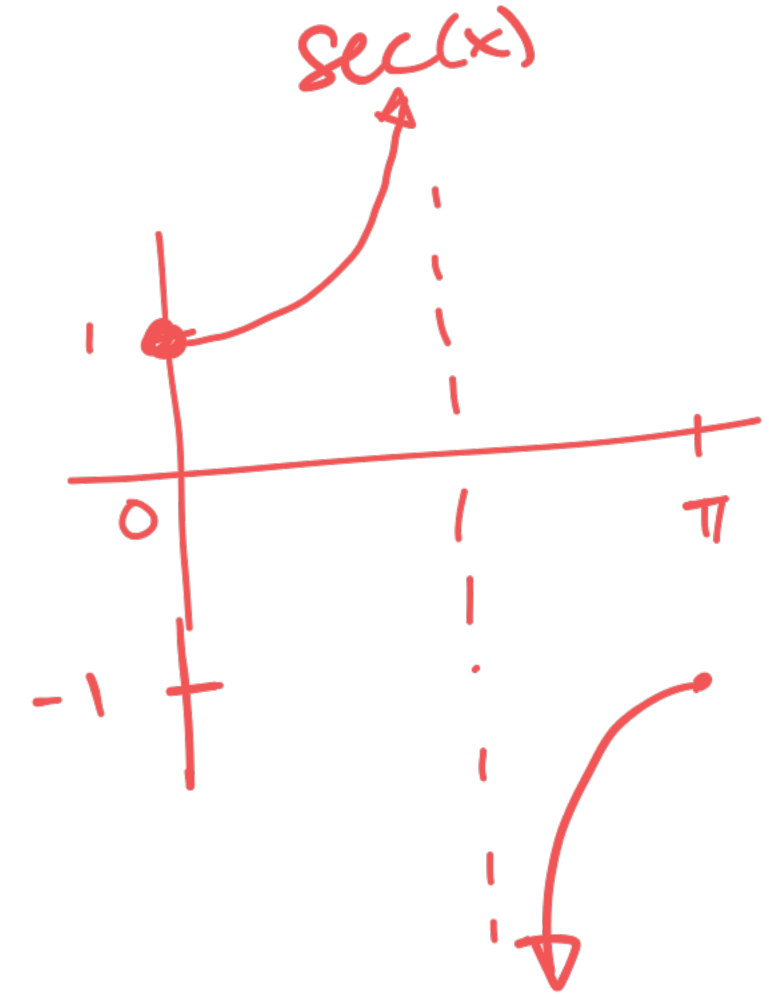
Example:

Consider $\displaystyle\int_0^{\pi} (\sec(x))^2\,dx = \tan(x)\Big|_0^{\pi} = \tan(\pi) - \tan(0) = 0$

$\sec(x) = \dfrac{1}{\cos(x)}$

$(\sec(x))^2$





Cannot apply FTC2 in this case!

because $(\sec(x))^2$ is not continuous on $[0, \pi]$!